

Friday 23 June 2023 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 A surface is defined in 3-D by $z = 3x^3 + 6xy + y^2$.

Determine the coordinates of any stationary points on the surface. [7]

- 2 A sequence is defined by the recurrence relation $4t_{n+1} - t_n = 15n + 17$ for $n \geq 1$, with $t_1 = 2$.

(a) Solve the recurrence relation to find the particular solution for t_n . [7]

Another sequence is defined by the recurrence relation $(n+1)u_{n+1} - u_n^2 = 2n - \frac{1}{n^2}$ for $n \geq 1$, with $u_1 = 2$.

(b) (i) Explain why the recurrence relation for u_n **cannot** be solved using standard techniques for non-homogeneous first order recurrence relations. [1]

(ii) Verify that the particular solution to this recurrence relation is given by $u_n = an + \frac{b}{n}$ where a and b are constants whose values are to be determined. [5]

A third sequence is defined by $v_n = \frac{t_n}{u_n}$ for $n \geq 1$.

(c) Determine $\lim_{n \rightarrow \infty} v_n$. [2]

- 3 A surface, S , is defined by $g(x, y, z) = 0$ where $g(x, y, z) = 2x^3 - x^2y + 2xy^2 + 27z$. The normal to S at the point $(1, 1, -\frac{1}{9})$ and the tangent plane to S at the point $(3, 3, -3)$ intersect at P.

Determine the position vector of P. [8]

- 4 The set G is given by $G = \{\mathbf{M}: \mathbf{M} \text{ is a real } 2 \times 2 \text{ matrix and } \det \mathbf{M} = 1\}$.
- (a) Show that G forms a group under matrix multiplication, \times . You may assume that matrix multiplication is associative. [5]
- (b) The matrix \mathbf{A}_n is defined by $\mathbf{A}_n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ for any integer n . The set S is defined by $S = \{\mathbf{A}_n : n \in \mathbb{Z}, n \geq 0\}$.
- (i) Determine whether S is closed under \times . [2]
- (ii) Determine whether S is a subgroup of (G, \times) . [2]
- (c) (i) Find a subgroup of (G, \times) of order 2. [2]
- (ii) By considering the inverse of the non-identity element in any such subgroup, or otherwise, show that this is the only subgroup of (G, \times) of order 2. [2]

The set of all real 2×2 matrices is denoted by H .

- (d) With the help of an example, explain why (H, \times) is **not** a group. [2]

- 5 The matrix \mathbf{P} is given by $\mathbf{P} = \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}$ where a is a constant and $a \neq 3$.
- (a) Given that the acute angle between the directions of the eigenvectors of \mathbf{P} is $\frac{1}{4}\pi$ radians, determine the possible values of a . [8]
- (b) You are given instead that \mathbf{P} satisfies the matrix equation $\mathbf{I} = \mathbf{P}^2 + r\mathbf{P}$ for some rational number r .
- (i) Use the Cayley-Hamilton theorem to determine the value of a and the corresponding value of r . [4]
- (ii) Hence show that $\mathbf{P}^4 = s\mathbf{I} + t\mathbf{P}$ where s and t are rational numbers to be determined. You should **not** calculate \mathbf{P}^4 . [3]

END OF QUESTION PAPER

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